Image Registration Using Lie Groups

Veronika Zimmer, Master's thesis

Abstract

Image registration is a very important area of medical image processing. Common image registration methods optimize an objective function over a space of transformations. One possible method is to use optimization procedures on Lie groups because most of the spatial transformation spaces do not form a vector space but a Lie group. These optimization methods are Newton methods. The use of the exponential map guarantees that these methods do not leave the Lie group (Fig. 1).

The main part of this thesis is the description of notions of differential geometry and the theory of Lie groups. On this basis the Newton methods on Lie groups used in Vercauteren et al. 2007 are presented. In addition the connection of these methods to the demons algorithm and its diffeomorphic extension is described (Vercauteren et al. 2009).



Fig. 1: In every iteration step of the Newton method on Lie groups a search direction s is computed and is projected back onto the Lie group \mathcal{G} through the exponential map exp.