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# Variational Image Registration Allowing for Discontinuities in the Displacement Field

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**Summary.** Registration of medical images is an active field of current research. The problem is to find a transformation which aligns two given images. The resulting displacement field may be described as a linear combination of pre-selected basis functions (parametric approach), or, as in our case, it may be computed as a minimizer of a functional (non-parametric or variational approach). This functional combines a similarity measure and a smoothness term. The first one puts the comparison of the images into quantifiable terms whereas the latter one regularizes the displacement field. The minimizing task is tackled by computing the Gâteaux derivative of the functional resulting in a set of nonlinear partial differential equations for the displacement field. These equations are linearized by means of a fixed-point iteration scheme and discretized by a standard finite difference approach.

A conventional variational method results in a globally smooth displacement field. However, a variety of clinical applications involve topology changes between the two images as for instance brain shift or tumor appearance or resection. For such applications a generalization of the standard method is needed which allows for localized discontinuities in the displacement field.

The variational image registration approach presented here assumes a segmentation of the images into corresponding subdomains. At the interfaces between neighbouring subdomains the influence of the smoothness term can be suppressed by introducing a spatially dependent weighting function. By choosing it appropriately this allows for opening or closing of a gap between image regions.

We demonstrate the capability of this new registration method by means of a one-dimensional synthetic example and a two-dimensional MR head image. However, our method can be applied to images of arbitrary dimensionality.

**Key words:** Image registration, finite difference method, variable regularization, discontinuities