Non-rigid image registration

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ABSTRACT

Image registration, in particular medical image registration, has been subject to extensive studies in the past years. Its versatile and important applications have attracted researchers from various branches, including numerical analysts. The goal of image registration is to find a transformation that aligns one image to another. To be successful, the registration scheme has to be adapted to the problem under consideration. Depending on the application at hand, it is often desirable to constrain the wanted deformation. The idea is to incorporate higher level information about the expected deformation. In this note, we examine some intuitive ways of adding user knowledge. Moreover we show, that all of these may be phrased in a variational setting allowing for high performance numerics. In addition, we present various instructive examples.

Keywords: Image registration, non-rigid, constraints

1. INTRODUCTION

Image registration is one of today's challenging image processing problems. The objective is to find a geometrical transformation that aligns points in one view of an object with corresponding points in another view of the same object or a similar one. Particularly in medical imaging, there are many instances that demand for registration. Typical examples include the treatment verification of pre- and post-intervention images, study of temporal series of images, and the monitoring of time evolution of an agent injection subject to a patient-motion. Another important area is the need for combining information from multiple images acquired using different modalities, sometimes also called fusion. Typical examples include the fusion of computer tomography (CT) and magnetic resonance (MR) images, of CT and positron emission tomography (PET), or of CT and ultrasound images (US). Image registration is inevitable whenever images acquired from different subjects, at different times, or from different scanners, need to be combined or compared for analysis or visualization. In the past two decades computerized image registration has played an increasingly important role in medical imaging (see, e.g. [11,9,12,14] and references therein).

Due to the wide range of applications a variety of different registration techniques has been developed. Here, we focus on so-called *intensity-driven approaches*. These schemes aim to match intensity patterns between a deformed scan and the target based on a rigorous mathematical criterion. Depending on the application, different strategies may be employed. From a practical point of view, it is desirable to incorporate properties of the underlying problem into the registration scheme. Here, we provide a toolbox of registration routines which enables the user to choose in a consistent way building blocks for schemes which cover a wide range of applications. The idea is to phrase each individual block in terms of a variational formulation. This not only allows for a unified treatment but also for fast and reliable implementation. The various building blocks comprises three categories: smoother and internal forces, distances and external forces, and "hard'" or "soft'" constraints. The *internal forces*, are defined for the wanted *displacement field* itself and are designed to keep the displacement field smooth during deformation. In contrast, the *external forces* are computed from the image data and are defined to drive the displacement in order to arrive at the desired registration result. Whereas the internal forces implicitly constrain the displacement to obey a smoothness criterion, the additional *constraints* force the displacement to satisfy explicit criteria, like for example landmark or volume preserving imposed constraints.

2. A UNIFIED APPROACH TO NON-LINEAR REGISTRATION

Let us start out with some notation. The key players are a reference image R and a template image T to be deformed. The images are represented by the compactly supported mappings $R, T : \Omega \to \mathbb{R}$, where without loss of generality,

$\Omega =]0,1[^d$. Hence, $T(x)$ denotes the intensity of the template at the spatial position x. Our approach is valid for images of any spatial dimension d, there is no restriction to $d = 2,3,4$. Given a reference image R and a template image T, the goal is to find a "reasonable" *transformation* $y: \mathbb{R}^d \to \mathbb{R}^d$, such that the deformed image $T[y]$ is "similar" to R.

There are various ways of computing a suitable transformation y. We focus on intensity based non-rigid registration approaches. The approach attempts to minimize an appropriate functional. It typically consists of two building blocks. The first is responsible for external forces, which are computed from the image data, whereas the second computes the internal forces, which are defined for the wanted displacement field itself. The internal forces are designed to keep the displacement field smooth during deformation, while the external forces are defined to obtain the desired registration result. The registration problem may be phrased as

$$J[y] := \mathcal{D}[R, T[y]] + \alpha\, \mathcal{S}[y] = \min, \qquad (1)$$

Here, $\mathcal{D}$ represents a *distance measure* (external force), whereas $\mathcal{S}$ denotes a *smoother* for y (internal force). The parameter α may be used to control the strength of the smoothness of the displacement versus the similarity of the images. The smoother $\mathcal{S}$ is also called *regularizing* term. This therm is unavoidable. Arbitrary transformations may lead to cracks, foldings, or other unwanted deformations. With an appropriate smoother it becomes possible to prefer particular transformations, which seem to be more likely than others. Moreover, the regularized problem becomes a starting point for a stable numerical implementation. The actual choice of $\mathcal{D}$ and $\mathcal{S}$ depends on the application under consideration.

From a numerical point of view, it is desirable that $\mathcal{D}$ and $\mathcal{S}$ possess a Gâteaux-derivative. For this case one may characterize a minimizer of $\mathcal{J}$ as solution of the so-called the Euler-Lagrange equations which are of the form

$$f(x, y(x)) + \alpha\, Ay(x) = 0, \ \text{for all}\ x \in \Omega \qquad (2)$$

and some additional boundary conditions. The so-called *force field* f is related to the Gâteaux-derivative of the distance measure $\mathcal{D}$ and the partial differential operator A is related to the Gâteaux-derivative of the smoother $\mathcal{S}$, respectively. The above semi-linear partial differential equation allows for a fast and robust computation of the wanted minimizer; for details see [12,6,7]. Alternatively, one may start out by discretizing the functional (1) ending up with a high dimensional non-linear optimization problem. Again, the ingredients of the gradient descent or Newton-type approach are related to the above mentioned force field and differential operator. For details, we refer to [10].

Distance measure One of the building blocks in equation (1) is the similarity criterion. As mentioned above we concentrate on those measures $\mathcal{D}$ which allow for differentiation. The most common choices for distance measures in image registration are the *sum of squared differences, cross correlation, cross validation*, and *mutual information.*

The sum of squared differences is probably the most popular distance measure. This measure is based on a point-wise comparison of image intensities:

$$\mathcal{D}^{\text{SSD}}[T[y], R] = \frac{1}{2}\int_\Omega (T(y(x) - R(x))^2 \,\mathrm{d}x.$$

The associated force field looks like

$$f(x, y) = (R(x) - T(y(x))\, \nabla T(y(x))).$$

This measure is often used when images of the same modality have to be registerd. For explicit formulae of the other distance measures see, e.g. [12,15,14].

Smoother The nature of the deformation depends strongly on the application under consideration. For example, a slice of a paraffin embedded histological tissue does deform elastically, whereas the deformation between the brains of two different individuals is most likely not elastic. Therefore, it is necessary to supply a model for the kind of the expected deformation. We now present some of the most prominent smoothers $\mathcal{S}$.

Elastic registration This particular smoother measures the *elastic* potential of the deformation. In connection with image registration it has been introduced by Broit [3] and discussed by various image registration groups; see e.g. [3,6]. Broit's approach is based on

$$\mathsf{S}^{\text{elas}}[y] = \int_\Omega \frac{\lambda+\mu}{2}|\text{div } y| + \frac{\mu}{2}\sum_{l=1}^{d}|\nabla y_l|^2 \,\text{d}x.$$

Here, λ and μ denote the so-called Lamé-constants. For this choice of regularizer, the Euler-Lagrange equations are nothing but the Navier-Lamé equations.

Fluid registration Due to the fact that an elastic body memorizes its non-deformed initial state (ruber band), elastic registration schemes are only able to compensate for small deformations. The situation changes for the viscous fluid model. Here the body adapts to its current state (honey) and consequently is much more flexible than an elastic body. The viscous fluid approach was introduced for image registration by [4]. The partial differential operator associated to this approach is again the Navier-Lamé operator, this time however, applied to the velocity of the deformation field. The wanted deformation is related to the velocity via the material derivative.

Since the viscous fluid approach is quite flexible, it is mainly used when the focus is more on similarity than on "natural" deformation process. For example, for the design of a probabilistic brain atlas, a biophysical model for the nature of the deformations is not available, but the fluid registration has been proven be a valuable tool.

Diffusion registration Fischer & Modersitzki introduced the *diffusion* regularization

$$\mathsf{S}^{\text{diff}}[y] = \sum_{l=1}^{d}\int_\Omega |\nabla y_l|^2 \,\text{d}x.$$

The associated Gâteaux-derivative leads to the well-studied Laplace-operator. This regularizer does penalize the derivative of the transformation and in addition does allow for a fast solution of the associated linear system via an AOS-based technique, which does have a linear complexity. If speed is the goal, this approach may be the method of choice [7].

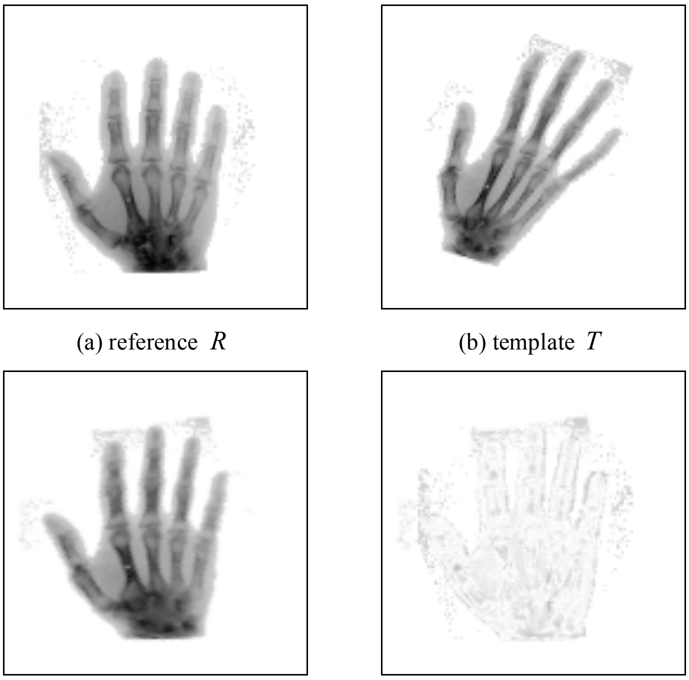
Curvature registration The regularizer

$$\mathsf{S}^{\text{diff}}[y] = \sum_{l=1}^{d}\int_\Omega |\Delta y_l|^2 \,\text{d}x.$$

is related to the *curvature* and was introduced by Fischer & Modersitzki [8]. The design principle behind the curvature smoother was the idea to make the non-linear registration phase more robust against a poor (affine linear) pre-registration. Since the smoother is based on second order derivatives, affine linear maps do not contribute to its costs. Therefore affine linear deformations are corrected naturally by the curvature approach. Again the Gâteaux-derivative is explicitly known and leads to the bi-harmonic operator.

2.1 An example: X-rays of hands

Just to give an example we present a registration result using the elastic regularizer. Fig. 1 shows two modified X-ray images of human hands, the reference (a) and the template image (b), and the elastically registered template (c). The figure also visualizes the difference between the result and the reference (d). We have chosen the Lamé-constants $\mu = 1$ and $\lambda = 0$. Evidently, the elastic registration reduces the image differences considerably.



(a) reference R (b) template T

(c) $T[y]$, elastically registered (d) difference image $|R - T[y]|$

Fig. 1 Two modified X-ray images of human hands (images from Y. Amit [0]). The result of the elastic registration of the reference image (a) and the template image (b) is illustrated in (c). The difference (d) between the elastically deformed template and the original is shown in (d).

3. ADVANCED APPROACHES

The framework described in Section 2 constitutes a flexible toolbox for non-linear image registration. It allows for a combination of various building blocks like SSD or mutual information as a distance measure with an elastic, fluid, diffusive, or curvature regularizer. The displacement is computed subject to a smoothness constraint. For example, in elastic matching, the constraint is given by a regularization based on the linear elastic potential of the displacement. In general, the constraint is applied globally with one global regularization parameter. Usually, the method provides satisfactory results due to the underlying physical model. Nonetheless, especially in medical applications there are cases for which the approach fails, since a global regularization does not allow for any local changes in the topology. In this section, we present add-ons to the basic approach that allows to overcome several specific shortcomings.

3.1 Weighted elastic image registration

A general image registration scheme is bound to fail when the images contain largely different high-contrast regions that are of no importance for the image analysis. A good registration may not be expected because the forces caused by the irrelevant structures will dominate the deformation. For example Fig. 2 shows CT-images of the abdomen. The images serve to plan a radiation therapy. The template image (d) has been acquired before some time before the therapy for planning purposes. The reference image (a) is taken just before the radiation treatment. The region of interest in the reference and the template image are shown in (b) and (e), respectively. Registration of both images is needed to compensate the different positions of the patient. Moreover, different intestinal fillings are responsible for significant differences of the images. Fig. 2 (c) shows the elastically registered template image. The deformation of the pelvic bone and neighboring tissues is unacceptable.

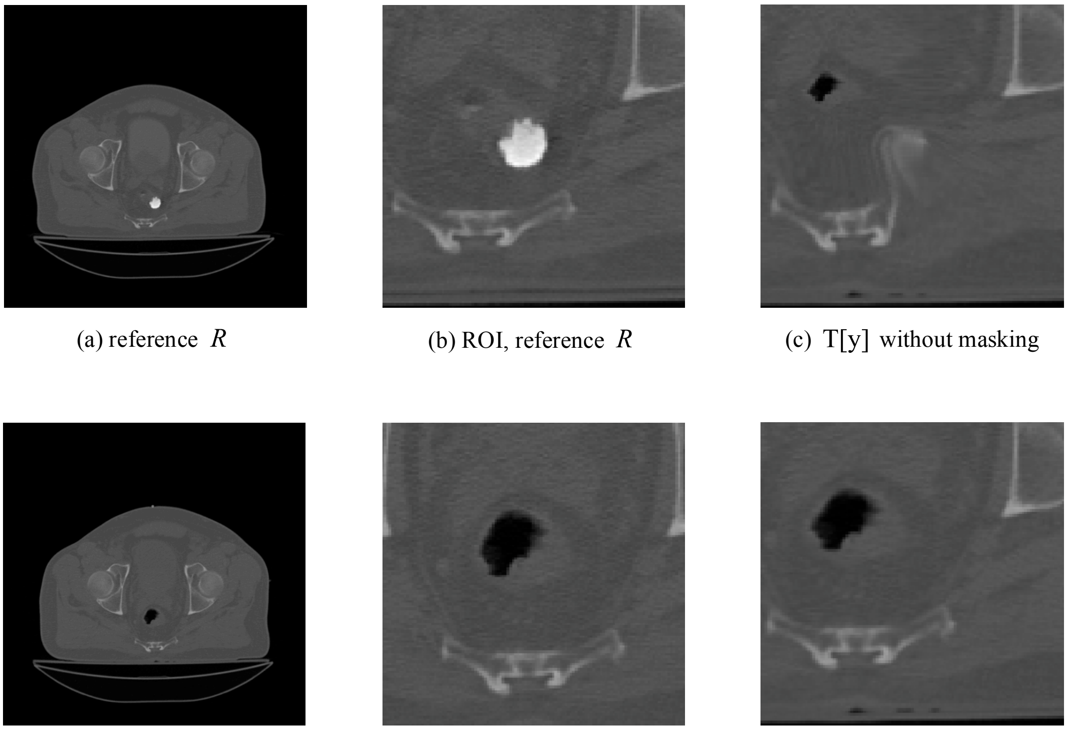
The problem may be cured by adapting the distance measure, see [5,17]. The idea is to hide the undesired features from the distance measure by weighting the images using a binary mask. Hence, given a segmentation B_T and B_R of the regions to be ignored in both the template and the reference image, we define

$$\mathcal{D}^{\text{Mask}}[T[y],R] = \frac{1}{2}\int_\Omega \left[\left(T(y(x)) - R(x) \right) \cdot M(y(x)) \right]^2 \mathrm{d}x ,$$

where

$$M(y(x)) = \begin{cases} 0 & \text{for } x \in (B_{T[y]} \cup B_R) \\ 1 & \text{otherwise} \end{cases} .$$

In effect, only those parts of the images that have neither been specified in B_T nor in B_R contribute to the distance value. The result of applying elastic image registration using the weighted difference measure is depicted in (f). The image shows that the shapes of both the pelvic bone and the neighboring tissues have been left largely intact by the image deformation.



(a) reference R (b) ROI, reference R (c) T[y] without masking

(d) template T (e) ROI, template T (f) T[y] with masking

Fig. 2 CT images for radiation therapy. The reference image (a) differs in the area of the abdomen from the template image (d). The region of interests of these images show (b) and (e). The elastically registered template image by using the unweighted distance measure is shown in (c) and by using the weighted distanve measure in (f).

3.2 Volume preserving registration

A typical application in medical imaging is to analyze images acquired before and after contrast administration. Unfortunately, the presence of the contrast agent poses a substantial difficulty with the registration of those images. Bright regions seem to enlarge during the wash-in phase of the radiopaque marker. This enhancement is due to contrast uptake but not to movement of the patient. Clearly, the task of image registration is to align only the differences caused by movements.

Fig. 3. illustrates MR images of a female breast. One image (a) is taken during the wash-in phase and the other (d) during the wash-out phase of the contrast medium. A comparison of these two images indicates a suspicious region in the upper part of the images. Furthermore, the glandular tissue appears larger in the latter picture due to the influence of the contrast agent. To deal with this phenomenon in the fusion process, we require that the volume of the bright regions highlighted by the squares must be preserved by the deformation.

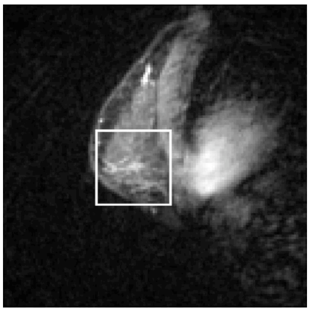
The idea is to supplement the general registration model by proper constraints $\mathcal{C}$. as proposed in [10]. A formulation of the problem now reads: Find a transformation y such that

$$\mathcal{D}[T[y],R] + \alpha\, \mathcal{S}[y] \xrightarrow{y} \min \quad \text{subject to} \quad \mathcal{C}[y] = 0, x \in \Omega,$$

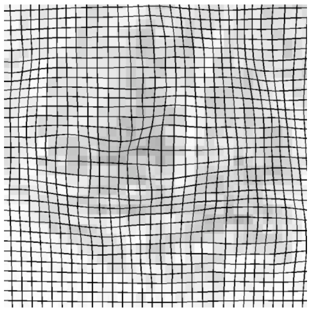
where

$$\mathcal{C}[y] = \det(\nabla y) - 1.$$

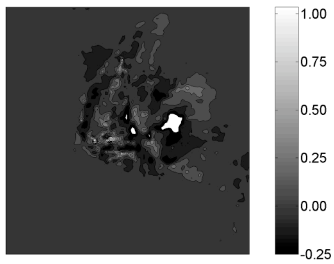
With respect to the difference measure alone, the unconstrained registration of the MR breast images, see Fig. 3, yields a very good result. However, the region marked by the square has been reduced so as to match the one in the reference image. The drastic change in volume is apparent in the visualization of the grid (b). In contrast, the constrained approach causes considerably less deformation in that region. Fig (c) and (f) show the pointwise map of change of volume. Using the unconstrained approach, we observe a considerable change of volume for the breast. For the constrained approach the change is almost zero.



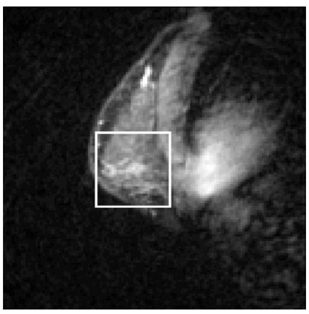
(a) reference R with ROI



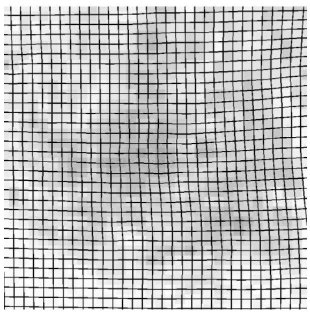
(b) ROI, difference with grid, no constraints



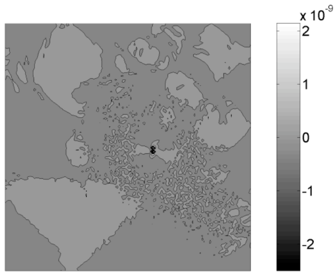
(c) change of volume, no constraints



(d) template T with ROI



(e) ROI, difference with grid, with constraints



(f) change of volume, constraints

Fig. 3 MR images of a female breast; images from Bruce Daniel, Lucas Center for Magnetic Resonance Spectoscopy ans Imaging, Standford University. The reference image (a) and the template image (b) differ in the region marked by a square. The unconstrained approach reduces this region, see (b), (c). By using the constrained approach, the volume of the breast is preserved, see (e), (f).

3.3 Registration and intensity correction

For many medical application, registration and intensity correction intertwine. For example in magnet resonance imaging (MRI) intensity inhomogeneties lead to various problems. See for example the MRI data of a human brain in Fig. 4. The reference image (a) and the template image (b) show on the one hand a mismatch of intensities. On the other hand there are small geometrical distortions. To solve both problems simultaneous, Modersitzki & Wirtz developed a methodological framework for joint registration and intensity correction (RIC), see [13]. The idea is to introduce a pointwise intensity correction $c = c(x)$. In order to rule out unwanted solutions c, they add an additional penalty or regularizer to the registration model (1). The final formulation of the joint registration and intensity correction problem reads:

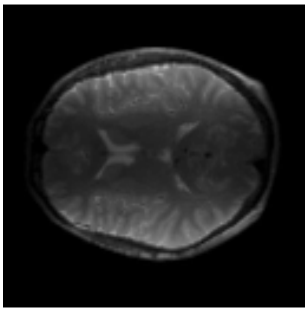
Given images R and T, find (y,c) such that

$$\mathcal{D}[T[y],cR] + \alpha\,\mathcal{S}[y] + \beta\mathcal{H}[c] \xrightarrow{y} \min.$$

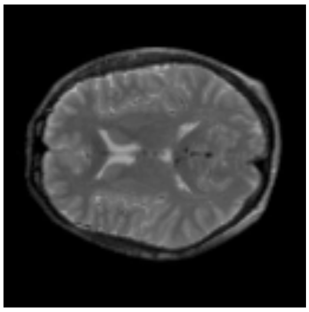
Here, α anb β are regularization parameters. The role of $\mathcal{H}$ is to balance between very smooth (no correction) or very rough (no registration). Modersitzki & Papenberg propose a total variation (TV) regularizer

$$\mathcal{H}[y] := \int_\Omega |\nabla c|\,\mathrm{d}x.$$

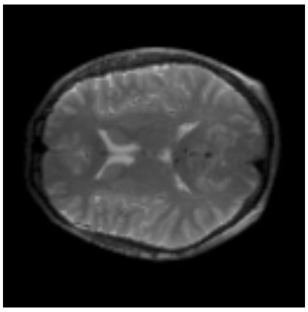
The example in Fig.4 focuses on intensity correction. The registration of the geometrical distortion is subordinate. We present results obtained by the RIC approach by using the SSD distance measure, the diffusion regularizer and $\alpha = 5000$ and $\beta = 10$.



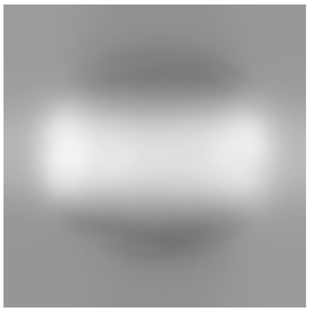
1. (a) reference R



(b) template T



(c) corrected reference cR



(d) correction c

Fig. 4 MR images of a brain with inhomogeneties (images from Samsonov et al [16]). SSD with diffusion and total variation regularization, $\alpha = 5000, \beta = 10$. The reference image (a) and the template image (b) show a mismatch of intensities. The correction of the reference by the correction field (d) provides the image (c).

A visual inspection of the images (a) and (b) indicates that the correction field c should be very smooth and regular. In fact, the results show a very small change in the geometry. The correction field is displayed in (d). With this correction we are able to compensate the intensity inhomogeneities of the refernce image, see (c). For the reduction we have $|T[y]-cR|/|T-R| \approx 32\%$.

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