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Optimal image registration with a guaranteed one-to-one point match

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Zusammenfassung. Automatic, parameter-free, and non-rigid registration schemes are known to be valuable tools in various (medical) image processing applications. Typically, these approaches aim to match intensity patterns in each scan by minimizing an appropriate distance measure. The outcome of an automatic registration procedure in general matches the target image quite good on the average. However, it may by inaccurate for specific, important locations as for example anatomical landmarks. On the other hand, landmark based registration techniques are designed to accurately match user specified landmarks. A drawback of landmark based registration is that the intensities of the images are completely neglected. Consequently, the registration result away from the landmarks may be very poor.
Here, we propose a framework for novel registration techniques which are capable to combine automatic and landmark driven approaches in order to benefit from the advantages of both strategies. We also propose a general, mathematical treatment of this framework and a particular implementation. The procedure computes a displacement field which is guaranteed to produce a one-to-one match between given landmarks and at the same time minimizes an intensity based measure for the remaining parts of the images.

1 Introduction

Two fundamental approaches are popular in todays image registration. One is based on the detection of a number of outstanding points, the so-called *landmarks*, and the second one is based on the minimization of an appropriate chosen *distance measure*. For the landmark based registration, a user has to identify a number of landmarks. Furthermore, he has to choose a *regularization term*, where typically the thin-plate-spline (TPS) regularizer is used; cf., e.g., [2], [9]. The distance based registration technique relies on two ingredients: one is a distance measure $\mathcal{D}$ and the other one a regularizer $\mathcal{S}$. The regularizer is needed since the problem is ill-posed; cf. e.g., [8].

Here, we propose a general framework for combination of landmark and distance measure based approaches. It is based on the minimization of a regularized distance measure subject to some interpolation constraints; see also [3] and, e.g., [4]. For ease of presentation, we focus on the spatial dimension three.

Given are the images $R, T : \Omega \subset \mathbb{R}^3 \to \mathbb{R}$, a regularizer $\mathcal{S}$, a distance measure $\mathcal{D}$, and the landmarks $r^j, t^j \in \Omega$, $j = 1, \ldots, m$. We are looking for a displacement $u = (u_1, u_2, u_3) : \mathbb{R}^3 \to \mathbb{R}^3$, such that

$$\mathcal{J}[u] = \min \quad \text{subject to} \quad u(t^j) = d^j := t^j - r^j, \quad j = 1, \ldots, m, \tag{1}$$

where $\mathcal{J}[u] := \mathcal{D}[R, T; u] + \alpha \mathcal{S}[u]$, and α is a regularization parameter. One may find various choices for the distance measure $\mathcal{D}$ and the regularizer (or smoother) $\mathcal{S}$ in the literature; see, e.g., [5] for an overview. To simplify the discussion and to demonstrate the performance of our new approach we have chosen here the commonly used so-called *sum of squared differences measure* and the so-called *curvature* smoother,

$$\mathcal{D}[R, T; u] \quad = \mathcal{D}^{\text{SSD}}[R, T; u] := \tfrac{1}{2} \int_\Omega \left(R(x) - T(x - u(x))\right)^2 dx, \tag{2}$$

$$\mathcal{S}[u] \quad = \mathcal{S}^{\text{curv}}[u] \quad := \tfrac{1}{2} \textstyle\sum_{\ell=1}^{3} \int_\Omega (\operatorname{div} u_\ell(x))^2 \; dx. \tag{3}$$

Without additional interpolation constraints, these choices lead to the well-known *curvature registration* approach; cf., [6].

2 Computing a solution

To compute a minimum of (1) we apply the calculus of variations, that is we compute the GÂTEAUX-derivative of the associated functional and subsequently seek for stationary points of the derivative. One finally ends up with the following necessary conditions for a minimizer (see [7] for details),

$$-f(x, u(x)) + \mathcal{A}[u](x) + \textstyle\sum_{j=1}^{m} \lambda_j \delta_{t^j} = 0, \quad \text{for all} \quad x \in \Omega, \tag{4}$$

$$\text{and} \quad \delta_{t^j}[u] - d^j = 0, \quad j = 1, \ldots, m, \tag{5}$$

where δ denotes the point-evaluation functional, $\delta_z[u] = u(z)$. This system of non-linear, distributional partial differential equations consists of three parts. More precisely, the so-called *force field* f results from the GÂTEAUX-derivative of the distance measure $\mathcal{D}$, the differential operator $\mathcal{A}$ results from the GÂTEAUX-derivative of the regularizer $\mathcal{S}$, and the δ-functionals are related to the interpolation constraints. For the particular choices of $\mathcal{D}$ and $\mathcal{S}$ introduced in eqn. (2) and (3), respectively, we have

$$f(x, u(x)) = (T(x - u(x)) - R(x)) \cdot \nabla T(x - u(x)), \tag{6}$$

$$A[u] = A^{\text{curv}}[u] = \alpha \Delta^2 u, \tag{7}$$

where Δ^2 is the vector-valued bi-harmonic operator.

In principle, this system may be solved in two steps. First, we compute a particular solution w and fundamental solutions v^j,

$$\mathcal{A}[w] = f(\cdot, u(\cdot)) \quad \text{and} \quad \mathcal{A}[v^j] = -\delta_{t^j}, \quad j = 1, \ldots, m. \tag{8}$$

Then the superposition

$$u = w + \sum_{j=1}^{m} \lambda_j v^j, \tag{9}$$

does solve the PDE (4) for any choice of the coefficients λ_j. These coefficients are now used to compute a solution u which on top does satisfy the interpolation conditions (5). From (5), we have

$$d_\ell^j = u_\ell(t^j) = w_\ell(t^j) + \sum_{i=1}^{m} \lambda_\ell^i v_\ell^i(t^j), \quad \ell = 1,2,3, \quad j = 1,\dots,m,$$

or, with $B^\ell := (v_\ell^i(t^j))_{j,i=1}^m$, $\lambda^\ell = (\lambda_\ell^i)_{i=1}^m$, $b^\ell = (d_\ell^j - w_\ell(t^j))_{j=1}^m$, we have $B^\ell \lambda^\ell = b^\ell, \quad \ell = 1,2,3$.

Altogether, u (cf. (9)) is by construction a stationary point for the functional $\mathcal{J}$, as required. However, the above outlined scheme is not applicable in the present form, as the solve for w in (8) requires the knowledge of the wanted solution u. To bypass this problem one may use a fixed-point type iteration. That is, starting with an initial guess $u^{(k)}$ fulfilling the interpolation constraints (5), we solve

$$\mathcal{A}[w^{(k+1)}] = f(\cdot, u^{(k)}(\cdot)) \quad \text{and update} \quad u^{(k+1)} = w^{(k)} + \sum_{j=1}^{m} \lambda_j v^j.$$

Another common approach to overcome the non-linearity in the force field f is based on a time-marching algorithm and can be used as well.

Two remarks are in order. First, we note that the adjustment of the parameters λ_j in (9) in each iteration step enforce a one-to-one match between the given landmarks, no matter at what point the iteration is stopped. Thus, we are able to guarantee the desired correspondence of anatomical landmarks.

Moreover, the introduction of interpolation constraints does not effect the complexity of the overall scheme as opposed to conventional schemes without interpolation constraints. The computation of the fundamental solutions v^j in (8) may be done once and forever, since they are independent of T and R and on the iteration index. In addition, for particular choices of $\mathcal{S}$, these solutions are known explicitly. The PDE associated with the fixed-point iteration or the time-marching scheme is identical to one obtained by solving the registration problem without additional landmarks. In other words, existing codes can be modified easily. For example, the ones outlined in [5] would lead to schemes of $\mathcal{O}(n \log n)$ or even $\mathcal{O}(n)$ complexity, depending on the chosen regularizer $\mathcal{S}$, where, n denotes the number of voxel. The overhead in the new approach is the solution of a linear system of the order m (where m denotes the number of landmarks) and the correction of u; cf. (9).

3 Example

In this section we present an example which does compare the new approach to the ones based solely on landmarks and the ones based on a non-rigid registration without landmarks. Figure 1 displays the registration of X-rays of a two human hands; images from [1]. The landmark based displacement has been computed using *thin-plate-splines*; cf., e.g., [2]. Note, that the landmarks are perfectly matched, whereas the rest of the hand is slightly "bent". The other computations are based on the distance measure and smoother introduced in (2) and 3), respectively; cf., e.g. [6]. The result after *curvature registration* looks perfect. However, a close examination shows that the landmarks are slightly off. Finally, we display several intermediate steps of the *curvature registration* with additional landmarks. It shows that not only the landmarks are perfectly matched but also the remaining part is nicely registered.

4 Conclusions

We have proposed a novel framework for parameter-free, non-rigid registration scheme which allows for the additional incorporation of user defined landmarks. It enhances the reliability of conventional approaches considerably and thereby their acceptability by practitioners in a clinical environment.

It has been shown that the new approach does compute a displacement field which is guaranteed to produce a one-to-one match between given landmarks and at the same time minimizes an intensity based measure for the remaining parts of the images. Moreover, its complexity is comparable to the one for a conventional registration scheme without additional landmarks. Finally, this approach may also be used to derive a good starting guess for the desired displacement, which may save computing time and may prevent a scheme for trapping into unwanted minima.

References

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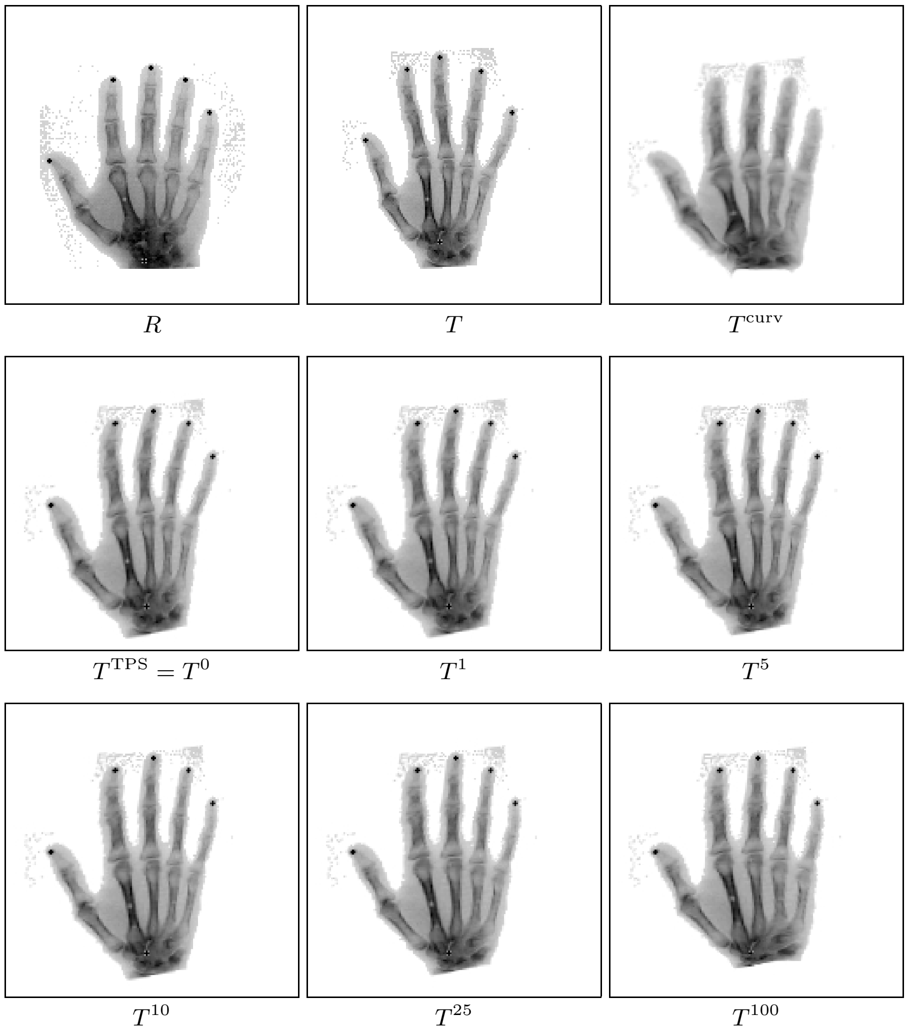

Abb. 1. Registration results for X-rays of a human hand (images from Y. AMIT [1]): TOP LEFT: reference R with 6 landmarks, TOP MIDDLE: template T with 6 corresponding landmarks, TOP RIGHT: template T^{curv} after distance measure based registration (after 100 iterations), T^k, $k = 0, 1, 5, 10, 25, 100$ intermediate results of the DLM (Distance and LandMark based) registration with 6 corresponding landmarks.