

# Improving an affine and non-linear image registration and/or segmentation task by incorporating characteristics of the displacement field

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## ABSTRACT

Image registration is an important and active area of medical image processing. Given two images, the idea is to compute a reasonable displacement field which deforms one image such that it becomes similar to the other image. The design of an automatic registration scheme is a tricky task and often the computed displacement field has to be discarded, when the outcome is not satisfactory. On the other hand, however, any displacement field does contain useful information on the underlying images.

It is the idea of this note, to utilize this information and to benefit from an even unsuccessful attempt for the subsequent treatment of the images. Here, we make use of typical vector analysis operators like the divergence and curl operator to identify meaningful portions of the displacement field to be used in a follow-up run. The idea is illustrated with the help of academic as well as a real life medical example. It is demonstrated on how the novel methodology may be used to substantially improve a registration result and to solve a difficult segmentation problem.

**Keywords:** medical image registration, registration based segmentation, characteristics of vector field, div, curl, local rigidity

## 1. INTRODUCTION

Image registration problem is one of the challenging tasks in medical image analysis. The aim of image registration is a reasonable displacement field between two images. The design of an automatic registration scheme is a tricky task and often the computed displacement field has to be discarded, when the outcome is not satisfactory. On the other hand, however, any displacement field does contain useful information on the underlying images. In this work we introduce a novel idea of extracting useful information from the displacement field obtained by a registration scheme. Based on this information we are able to compute a segmentation of the image. Furthermore, the segmentation can be utilized for an improved subsequent registration.

We start by defining the image registration problem. Given a reference image  $R$  and a template image  $T$  we wish to find a smooth transformation  $y : \mathbb{R}^d \rightarrow \mathbb{R}^d$  for the template such that  $R$  and  $T(y)$  become similar. After performing an initial registration, we analyze the resulting deformation  $y$ . To this end, we extract characteristic features from the deformation field. These features are generally modeled as a scalar field  $P(y)$ . In particular, here we consider the curl and the divergence of the deformation field, which appears to be a natural choice. Nevertheless, our approach is general and can also be used with other characteristics. After having computed such a characteristic  $P(y)$ , it is used to label or segment the underlying domain  $\Omega$ . That is, we compute a family of disjoint subsets  $\Sigma_j \subset \Omega$  with similar characteristics that form a partition of  $\Omega$ . The identified sets can be

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used to compute a possibly difficult segmentation of the images (see Example 1) and furthermore to improve the registration itself (see Example 2).

The remainder of this paper is organized as follows. Section 2 describes the non-linear registration problem, the characteristics of the deformation field used here and our idea to utilize the information from the displacement field for computing a segmentation of the image and for an improved subsequent registration. The experimental results are presented in Section 3. Finally, conclusions can be found in Section 4.

## 2. METHODS

Our approach consists of two major steps. First we perform an initial registration to compute a deformation field. Then we extract features from the resulting deformation that are used for a segmentation. Furthermore, we can use the segmentation in an optional third step for a subsequent improved registration. We start by describing the underlying non-linear registration scheme.

### 2.1 Registration

Here we use non-linear registration for the alignment of the reference  $R$  and template  $T$ . The images are modeled as continuous scalar fields, i.e.,  $R, T : \mathbb{R}^d \rightarrow \mathbb{R}$  where  $d = 2, 3$  denotes the image dimension. The registration task is performed by solving the following unconstrained optimization problem

$$J(y) := D(y; R, T) + \alpha S(y) \rightarrow \min. \quad (1)$$

Here,  $D$  is a distance measure describing the similarity between the reference image  $R$  and the deformed template image  $T(y) := T \circ y$ ,  $S$  is a regularizer forcing smoothness of the deformation field,  $\alpha > 0$  weights both building blocks against each other.

The specific choice of the distance measure is not in the focus of this work (for an overview see<sup>1</sup>). In our examples we used the sum of squared differences (SSD) distance measure

$$D(y; R, T) = \frac{1}{2} \|T(y) - R\|_{L_2(\Omega)}^2. \quad (2)$$

For the regularizer  $S$  we choose the so-called elastic regularizer<sup>2</sup>

$$S(y) = \frac{\mu}{2} \|\nabla \times y\|_{L_2(\Omega)}^2 + \frac{2\mu + \lambda}{2} \|\nabla \cdot y\|_{L_2(\Omega)}^2 \quad (3)$$

where  $\mu$  and  $\lambda$  are so-called Lamé-constants describing the elasticity of the underlying material. Typical values for  $\mu$  and  $\lambda$  are  $\mu = 1$  and  $\lambda = 0$ .

Next, we present an idea for using the information of curl and divergence of the displacement field. Typically, the outcome of a registration is a smooth deformation yielding a more or less perfect alignment of the reference and the template.

If the obtained registration result turns out to be non-satisfactory, it would be a pity to simply reject it. It appears to be a good idea, to learn from the characteristics  $P[y]$  of the displacement field in order to improve the registration. Possible examples of these characteristic features are the divergence and the curl of the displacement field.

### 2.2 Characteristics of the Deformation Field

As mentioned above, we use the divergence and the curl as characteristic features, i.e., we consider

$$P^{\text{curl}}(y) := \|\nabla \times y\|$$

and

$$P^{\text{div}}(y) := |\nabla \cdot y|.$$

The curl describes the turbulence of a body which "co-swims" in the flow of the vector field. It will be used to identify regions where the displacement field is nearly rigid. The divergence describes the local expansion or source strength of a certain area, thus it may be used to identify a local scaling or shearing behavior of the transformation.

Based on these characteristic features we compute a family of sub-domains  $\Sigma_j \subset \Omega$  that form a partition of  $\Omega$  in the sense

$$\Sigma_j \cap \Sigma_k = \emptyset \text{ iff } j \neq k \quad \text{and} \quad \bigcup_j \bar{\Sigma}_j = \bar{\Omega}, \quad (4)$$

where  $\bar{\Omega}$  denotes the closure of  $\Omega$ . This is done by a segmentation of  $P(y)$ . The idea is to find regions of similar characteristic, that is we require

$$P(y)(x) \approx \text{const} \quad \text{for } x \in \Sigma_j.$$

Furthermore, we do not want the sub-domains  $\Sigma_j$  being too small. To this end we additionally require

$$\text{vol}(\Sigma_j) := \int_{\Sigma_j} dx \geq \nu$$

where  $\nu > 0$  is a user prescribed threshold. The computed region  $\Sigma_j$  are then used to improve the registration and segmentation results. Whereas the regions  $\Sigma_j$  may be directly used for the labeling of the template image, their use in the registration task is not that obvious. Here, we present just one possible approach. The idea is to be utilize the  $\Sigma_j$  in order to subdivide the template and then to perform an individual and independent registration on these sets. Afterword the individual deformation fields are combined to yield an overall displacement field. The overall scheme is outlined in algorithmic form in Table 1.

STEP	ACTION	RESULT
1.	Compute affine and non-linear image registration.	$\rightarrow y$
2.	Calculate the characteristics of the displacement field.	$\rightarrow P[y]$
3.	Segment the images based on the calculation in step 2.	$\rightarrow \Sigma_j$
	<i>In case of a segmentation task, STOP.</i>	
4.	Compute separate registration for each segments.	$\rightarrow y_j$
5.	Combine the displacement fields.	$\rightarrow y_{new}$
	<i>If registration satisfying, STOP otherwise go to 2.</i>	

Table 1. An algorithm in which the non-linear image registration of certain information can be extracted and how this knowledge to the registration and/or segmentation can be successfully applied.

### 3. RESULTS

To demonstrate the performance of the outlined scheme, we present two examples based on standard problems in medical image processing. The first one concerns a situation where a standard segmentation routine is bound to fail. The second example deals with a registration problem, where a straightforward application inevitable produces unsatisfactory results.

### 3.1 An academic example

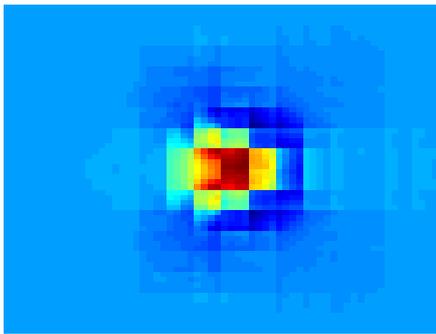
The first experiment mimics the follow-up segmentation of a tumor which is glued to a nearby vessel, as may be frequently seen for lung cancer.<sup>3</sup> A segmentation of the tumor based solely on the image information seems to be impossible, see Fig. 1(a) and (b).



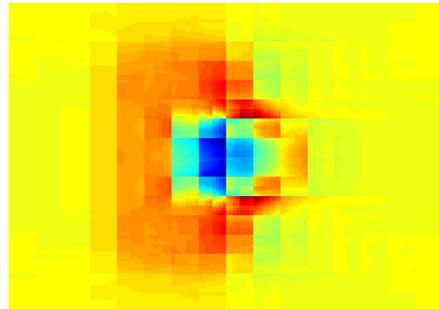
(a) tumor and vessel before treatment



(b) tumor and vessel after treatment



(c) divergence for the registration from (a) to (b)



(d) divergence for the registration from (b) to (a)

Figure 1. Idealized follow-up study of lung cancer: (a) before treatment, (b) after treatment, (c) divergence of the displacement field for the registration with (a) as a template and (b) as a reference image, (d) divergence of the displacement field for the registration with (b) as a template and (a) as a reference image.

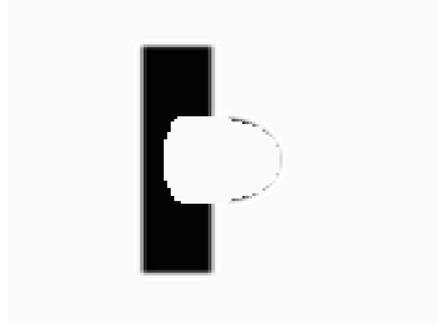
To solve this segmentation problem, we first compute a non-linear registration with  $\alpha = 0.01$ ,  $\lambda = 1$  and  $\mu = 1$  in both directions and subsequently compute the divergence of the obtained displacement fields. As it is apparent from Fig. 1(c) and (d) regions with a shrinking or expanding tumor are nicely detected. Finally, the acquired information is used in conjunction with the Chan-Vese approach<sup>4</sup> to obtain a segmentation. The results of segmentation of both images are depicted in Fig. 2. The nearly perfect segmented tumors are shown in the first column. The vessels without tumors are displayed in the second column.

### 3.2 A clinical example

The next experiment deals with a real life example. Here, it is illustrated on how one may improve the registration of two MR images of a knee, see Fig. 4 (a) and Fig. 4 (b). The registration of such images is important for the knee pre-surgical planning.<sup>5</sup> To start with, we computed a non-linear registration with respect to the parameter



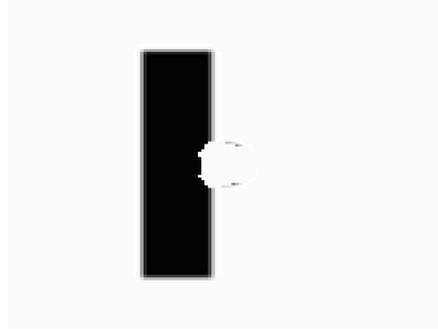
(a) tumor



(b) vessel



(c) tumor



(d) vessel

Figure 2. Results of the tumor and vessel segmentation with the introduced approach.

set  $\alpha = 1$ ,  $\lambda = 1$ ,  $\mu = 1$ . One may observe two problems in the obtained results (Fig. 4 (c)). The first one concerns the fact that the transformed image still displays a partly bent knee, while in the reference a stretched knee is to be seen. The second observation is an unnaturally deformed shinbone in the lower part of the transformed image. Former attempts to solve these problems are connected to the detection of the bones from an expert.<sup>6-8</sup> Next, we applied our new methodology, that is, we computed the curl of the displacement field discriminating regions of local movement and used this information to automatically split the reference and template image into two regions by computing a segmentation of the 'curl image', see Fig. 5. Hereafter the obtained regions undergo an independent affine linear registration step. The resulting displacement fields  $y_{new}$  are composed with the help of a well-known blending technique,<sup>9</sup> to obtain a composed displacement field for the whole image

$$y_{new}(x) = \gamma_1(x) \cdot y_1(x) + \gamma_2(x) \cdot y_2(x), \quad (5)$$

where  $\gamma_1, \gamma_2 : \mathbb{R}^d \rightarrow \mathbb{R}$  are two weight functions with  $\gamma_1(x) + \gamma_2(x) = 1$ , for all  $x$ . Typical example of such a weight function is shown in Fig. 3. For this example, we picked  $\epsilon = 10$ . Finally, the template image is transformed by this displacement field. The result is shown in Fig. 4 (d). It is easy to recognize that the aforementioned problems are no longer present.

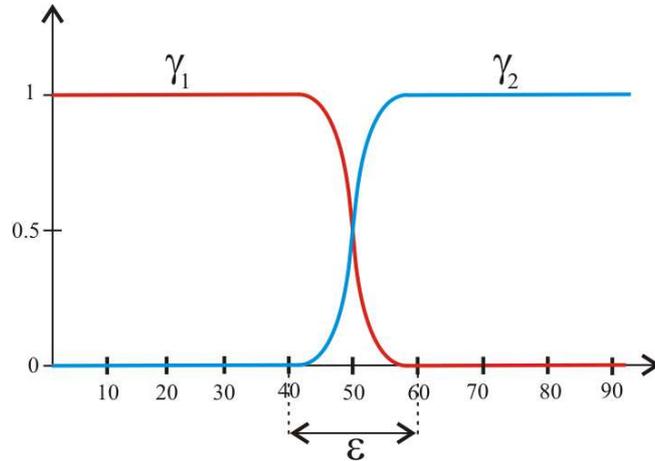


Figure 3. Typical example for blending functions. Here  $\epsilon$  defines a confidence region.

#### 4. CONCLUSIONS

This work shows that even an unsuccessful registration attempt may be used to advantage using a careful vector analysis of the computed displacement field to reveal characteristics which then may be used to improve the registration. The idea was successfully demonstrated for highly nontrivial registration and segmentation problems. The presented problems are example for two specific, tricky registration/segmentation tasks. It is planned to identify more problems which may benefit from the novel idea outlined in the paper.

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#### REFERENCES

- [1] Modersitzki, J., [*Numerical methods for image registration*], Oxford University Press (2003).
- [2] Broit, C., *Optimal registration of deformed images*, PhD thesis, Computer and Information Science, University of Pennsylvania (1981).
- [3] Savai, R., Wolf, J. C., Greschus, S., Eul, B. G., Schermuly, R. T., Hnze, J., Voswinckel, R., Langheinrich, A. C., Grimminger, F., Traupe, H., Seeger, W., and Rose, F., “Analysis of tumor vessel supply in lewis lung carcinoma in mice by fluorescent microsphere distribution and imaging with micro- and flat-panel computed tomography,” *Am J Pathol*, 937946 (2005).
- [4] Chan, T. and Vese, L., “An active contour model without edges,” *Scale-Space, Lecture Notes in Computer Science* (1999).
- [5] Komistek, R., Kane, T., and Mahfouz, M., “Knee mechanics: a review of past and present techniques to determine in vivo loads.,” *Journal Biomech* **38**, 215–228 (2005).
- [6] Little, J., Hill, D.-L. G., and Hawkes, D. J., “Deformation incorporating rigid structures,” *Computer Vision and Image Understanding* (66(2)), 223–232 (1997).
- [7] Loeckx, D., Maes, F., Vandermeulen, D., and Suetens, P., “Nonrigid image registration using free-form deformations with a local rigidity constraint,” in [*Medical Image Computing and Computer-Assisted Intervention (MICCAI), Lecture Notes in Computer Science*], 639–646 (2004).
- [8] Modersitzki, J., “Flirt with rigidity—image registration with a local non-rigidity penalty,” *Int. J. Comput. Vision* **76**(2), 153–163 (2008).



(a) reference image



(b) template image



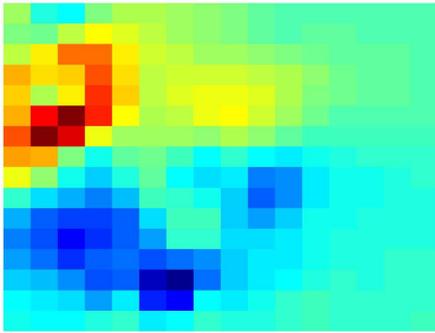
(c) plain registration



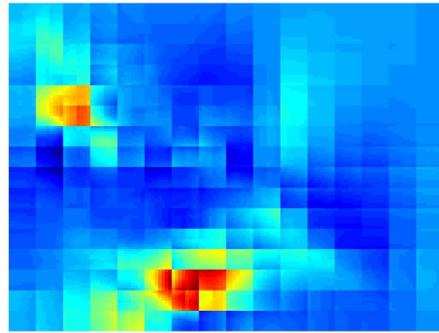
(d) new method

Figure 4. Results of the non-linear knee registration with old and new approach: (a) knee in the straight position as a reference image, (b) knee in the bent position as a template image, (c) result of a plain registration, (d) result of the introduced approach.

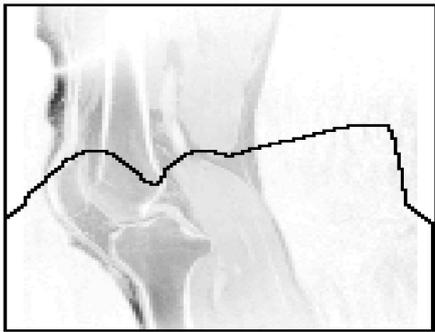
- [9] Porter, T. and Duff, T., “Compositing digital images,” in [*SIGGRAPH ’84: Proceedings of the 11th annual conference on Computer graphics and interactive techniques*], 253–259, ACM, New York, NY, USA (1984).



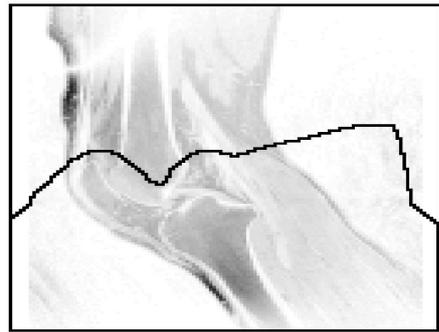
(a) curl



(b) divergence



(c) segmentation of  $R$



(d) segmentation of  $T$

Figure 5. (a) curl map of the displacement field of the plain registration, (b) divergence map of the displacement field of the plain registration (c) curl based segmentation of the reference image, (d) curl based segmentation of the template image